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ELEMENTS

OF

ARITHMETIC,

ADAPTED TO THE USE OF
COMMON SCHOOLS AND ACADEMIES.

PART I.

BY E. JONES, A.M.
[illegible] OF [illegible]
[illegible] COLLEGE, [illegible]

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HARVARD COLLEGE
LIBRARY

ELEMENTS

OF

ARITHMETIC.

PART I.

ADAPTED TO THE USE OF

COMMON SCHOOLS AND ACADEMIES.

BY Z. JONES, A. M.

PRINCIPAL OF ROCKINGHAM ACADEMY.

HAMPTON FALLS, N. H.

EXETER:

PRINTED BY F. GRANT.

1845.

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PREFACE.

To produce a Text Book for my own use and convenience, I have undertaken, in the following pages, to arrange the elementary processes of arithmetic in mathematical order, distinct from the more mixed and difficult operations; divest definitions and explanations of Latin terms; refer each process and explanation to a fundamental formula; classify subjects according to their relations to each other, and thus aid, in some small degree, in rendering the study of numbers a source of mental discipline and pleasure.

Hampton Falls, July 1845.

ARITHMETIC.

Arithmetic is that branch of the Mathematics, which treats of numbers. The characters used in arithmetical operations are 1, 2, 3, 4, 5, 6, 7, 8, 9, and are called figures. There is another used, called cipher, or zero, to denote position, having no numerical value.

§ 1. Values.

Every figure represents a determinate value. The value of 1 is one; of 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine. This determinate value of figures may be called their absolute value. Every figure may have also a relative value. When figures represent things of only one kind, their relative value depends upon *position*, and is called *simple* value. When figures represent things of different kinds, their value depends upon *position* and *kind*, and is called *compound* value. In simple values, the first position is unity, the second represents a value ten times as large as the first, the third, a value ten times as large as the second, and so on. This law of increase in simple relation is illustrated in the following Scale.

Position.	Rate of increase.	SCALE.
19th	×	Trillions
18th	×	Hundreds of thousands of billions
17th	×	Tens of thousands of billions
16th	×	Thousands of billions
15th	×	Hundreds of billions
14th	×	Tens of billions
13th	×	Billions
12th	×	Hundreds of thousands of millions
11th	×	Tens of thousands of millions
10th	×	Thousands of millions
9th	×	Hundreds of millions
8th	×	Tens of millions
7th	×	Millions
6th	×	Hundreds of thousands
5th	×	Tens of thousands
4th	×	Thousands
3d	×	Hundreds
2d	×	Tens
1st	×	Units

From Trillions, the Scale may be extended to Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, et cet.

Examples for Reading.

11.
134.
604892.
4383.
67104210.
567124.
167814678 92.
4121678.
164216891467 89.
8476541210198765454.
1467814678.
678912346789089.

Examples for Writtng.

Two. Twenty. Two hundred. Three hundred and fifty. Six hundred and seventy five. One thousand and twenty five. Thirteen millions and sixteen. One trillion, six billions, and eighty five.

In United States money, the relation of kinds is the same as in simple values. This is seen in the Table of U. S. money.

10 mills make 1 cent.
10 cents make 1 dime.
10 dimes make 1 dollar.
10 dollars make 1 eagle.

In accounts, dimes are included in the ten's place of cents. Dollars are separated from cents by a point, (,). When only one figure is used to express cents, care must be had to write it in unit's place of cents, and insert the mark of position (0) in the ten's place. Mills are written at the right of cents. The letters U. S. are contracted, thus, $, to denote United States money.

Examples for Writing.

Write under each other, according to their relative value, the following examples. One cent and four mills. Twelve cents and seven mills. One dollar, twelve cents and five mills. Twenty five dollars, ten cents and five mills.

§ 2. Uniting Simple Values.

When simple values are represented by more than one set of figures, it is often necessary to find their value, expressed in one set of figures. This process is called uniting simple values, or simple addition. Thus, 10 and 12 represent two values in two sets of figures; and 22 represent the same values in one set of figures. When values are thus united, the result is called sum, or amount. The sign of this process is a horizontal line, crossed by a perpendicular. (+)

To unite large values, write them under each other according to their position, those having the same relative value under each other, and unite the values in each column.

1. Unite 232, 421, 110, 654, 533.

232
421
110
654
533
———
10
14
18
———
1950 Amount.

2. Unite 321, 4025, 63478, 7012091.
3. Unite 14678, 24675, 46789, 5676.
4. Unite $ 2,06; $ 12,034; $ 25,026.
5. Unite $ 4,678; $ 16,084; $ 41,005; $ 250.

§ 3. Finding the difference between Simple Values.

When two equal values are compared, the difference between them is found to be nothing. Thus, the difference between 2 and 2 is nothing. But when two unequal values are compared, there is found a difference between them. Thus, between the two unequal values, 5 and 3, there is a difference of 2. This difference is expressed by the number, which shows how much larger one number is than the other. The sign of finding this difference is a horizontal line. (—)

To find the difference between large values, place the less value under the greater, as in § 2, and take the difference between the values in each position.

1. Find the difference between 848 and 424.

848
424
———
424 Difference.

2. Find the difference between 749 and 636.
3. Find the difference between 636 and 524

When a figure in the lower number is greater than the one above it, add 10 to the upper figure, and from that amount take the lower figure. In this case, add 1 to the next left hand figure in the lower number, before taking it from the one above it.

4. Find the difference between 234 and 175. Ans. 59.
5. Find the difference between 456 and 387. Ans. 69.
6. Find the difference between $125 and $75. Ans. $50.

This process is also called Simple Subtraction.

The greater number is called the Minuend, and the smaller, the Subtrahend. The operation may be verified, by uniting the difference and smaller value. If their amount agree with the greater value, the work is right.

7. Find the difference between $1000 and $1260.
8. Find the difference between $63,25 and $54,125.

§ 4. Repeating Simple Values.

Values repeated from 1 to 12.

2 times 1 are 2; 2 times 2 are 4; 2 times 3 are 6;
2 times 4 are 8; 2 times 5 are 10; 2 times 6 are 12;
2 times 7 are 14; 2 times 8 are 16; 2 times 9 are 18;
2 times 10 are 20; 2 times 11 are 22; 2 times 12 are 24;
3 times 1 are 3; 3 times 2 are 6: 3 times 3 are 9;
3 times 4 are 12; 3 times 5 are 15; 3 times 6 are 18;
3 times 7 are 21; 3 times 8 are 24; 3 times 9 are 27;
3 times 10 are 30; 3 times 11 are 33; 3 times 12 are 36;
4 times 1 are 4; 4 times 2 are 8; 4 times 3 are 12;
4 times 4 are 16; 4 times 5 are 20; 4 times 6 are 24;
4 times 7 are 28; 4 times 8 are 32; 4 times 9 are 36;

4 times 10 are 40; 4 times 11 are 44; 4 times 12 are 48; 5 times 1 are 5; 5 times 2 are 10; 5 times 3 are 15; 5 times 4 are 20; 5 times 5 are 25; 5 times 6 are 30; 5 times 7 are 35; 5 times 8 are 40; 5 times 9 are 45; 5 times 10 are 50; 5 times 11 are 55; 5 times 12 are 60; 6 times 1 are 6; 6 times 2 are 12; 6 times 3 are 18; 6 times 4 are 24; 6 times 5 are 30; 6 times 6 are 36; 6 times 7 are 42; 6 times 8 are 48; 6 times 9 are 54; 6 times 10 are 60; 6 times 11 are 66; 6 times 12 are 72;—7 times 1 are 7; 7 times 2 are 14; 7 times 3 are 21; 7 times 4 are 28; 7 times 5 are 35; 7 times 6 are 42; 7 times 7 are 49; 7 times 8 are 56; 7 times 9 are 63; 7 times 10 are 70; 7 times 11 are 77; 7 times 12 are 84;—8 times 1 are 8; 8 times 2 are 16; 8 times 3 are 24; 8 times 4 are 32; 8 times 5 are 40; 8 times 6 are 48; 8 times 7 are 56; 8 times 8 are 64; 8 times 9 are 72; 8 times 10 are 80; 8 times 11 are 88: 8 times 12 are 96:—9 times 1 are 9; 9 times 2 are 18; 9 times 3 are 27; 9 times 4 are 36; 9 times 5 are 45; 9 times 6 are 54; 9 times 7 are 63; 9 times 8 are 72; 9 times 9 are 81; 9 times 10 are 90; 9 times 11 are 99; 9 times 12 are 108;—10 times 1 are 10; 10 times 2 are 20; 10 times 3 are 30; 10 times 4 are 40: 10 times 5 are 50; 10 times 6 are 60; 10 times 7 are 70; 10 10 times 8 are 80; 10 times 9 are 90; 10 times 10 are 100; 10 times 11 are 110; 10 times 12 are 120;—11 times 1 are 11; 11 times 2 are 22; 11 times 3 are 33; 11 times 4 are 44; 11 times 5 are 55; 11 times 6 are 66; 11 times 7 are 77; 11 times 8 are 88; 11 times 9 are 99; 11 times 10 are 110; 11 times 11 are 121; 11 times 12 are 132;—12 times 1 are 12; 12 times 2 are 24; 12 times 3 are 36; 12 times 4 are 48; 12 times 5 are 60; 12 times 6 are 72; 12 times 7 are 84; 12 times 8 are 96; 12 times 9 are 108; 12 times 10 are 120; 12 times 11 are 132; 12 times 12 are 144.

This process is called Multiplication. The number repeated is called Multiplicand, and the number showing how many repetitions are to be made is called Multiplier. The result is called Product.

When values exceeding 12 are to be repeated, place them under each other as in § 2, and repeat each figure in the multiplicand as many times as there are units, tens, hundreds, &c. in the multiplier. Place the figures in the result under each other according to their relative value, and unite them as in § 2.

1. Multiply 25 by 5.

$$\begin{array}{rrr} & 2 & 5 \\ & & 5 \\ \hline & 2 & 5 \\ 1 & 0 & \\ \hline 1 & 2 & 5 \end{array} \text{ Product.}$$

2. Multiply 265 by 25

$$\begin{array}{rrrr} & 2 & 6 & 5 \\ & & 2 & 5 \\ \hline & & 2 & 5 \\ & 3 & 0 & \\ 1 & 0 & & \\ & 1 & 0 & \\ 1 & 2 & & \\ 4 & & & \\ \hline 6 & 6 & 2 & 5 \end{array} \text{ Product.}$$

3. Multiply 645 by 35.

$$\begin{array}{rrrrr} & & 6 & 4 & 5 \\ & & & 3 & 5 \\ \hline & & & 2 & 5 \\ & & 2 & 0 & \\ & 3 & 0 & & \\ & & 1 & 5 & \\ & 1 & 2 & & \\ 1 & 8 & & & \\ \hline 2 & 2 & 5 & 7 & 5 \end{array} \text{ Product.}$$

This process may be abbreviated by uniting some of the values during the process of repeating.

Third Example Abbreviated.

$$\begin{array}{rrrrr} & & 6 & 4 & 5 \\ & & & 3 & 5 \\ \hline & 3 & 2 & 2 & 5 \\ 1 & 9 & 3 & 5 & \\ \hline 2 & 2 & 5 & 7 & 5 \end{array} \text{ Product.}$$

To verify any operation of this kind, perform the work in the full and abbreviated form.

4. Multiply 2689 by 398.
5. Multiply $225,555 by 56.
6. Multiply 849 by 67. 7. Multiply $26,375 by 5.

The multiplier and multiplicand, taken together are called factors. And any numbers bearing the same relation to each other, are called factors. When any number contains two numbers, having relation as factors, it is called a composite number. If such a number be a multiplier, the repetition may be made, by multiplying by one factor, and that product by the other.

8. Multiply 84 by 15 (a composite number.)
9. Multiply 112 by 63 (a composite number.)
10. Multiply 224 by 49.

When there are ciphers at the right hand of the figures, annex them to the right hand of the product of the figures.

11. Multiply 870 by 800. 12. Multiply 600 by 500.

When the multiplier is 1 with ciphers annexed, annex the ciphers to the multiplicand.

13. Multiply 288 by 100. 14. Multiply 44 by 10000.

§ 5. Finding how many times one simple value is contained in another simple value.

The number, showing how many times a smaller value must be repeated to equal a larger one, indicates how many times the smaller value is contained in the larger one. This number is called the Quotient. The sign of this process is a horizontal line between two dots. (÷)

How many times are 2 contained in 4? 2 in 8? 2 in 10? 2 in 12? 3 in 6? 3 in 9? 3 in 12? 3 in 15? 4 in 8? 4 in 12? 4 in 16? 4 in 20? 4 in 24? 4 in 28?

When large values are contained in other large values, the following method is taken to find the quotient.

Find how many times 25 are contained in 155550?

25)155550(6222 Quotient.
150
——
55
50
——
55
50
——
50
50

The number contained in the other is called divisor. The number containing the divisor is called the dividend. The process of finding the quotient, is also called division.

When the divisor is less than 12, the following abbreviated form may be adopted.

Find how many times 8 are contained in 128824?

8)128824
——
16103 Quotient.

1. Divide 576 by 4. Ans. 144.
2. Divide 4284 by 6. Ans. 714.

To verify a process in division, multiply the quotient by the divisor, and to the product add the remainder, (if there be any,) and if the result agree with the dividend the work is right.

3. Divide 8961 by 124. 4. Divide 61786 by 36.

When the divisor has a cipher at its right, cut it off, and cut off one place from the right of the dividend, and proceed as before with the remaining figures. If there are more than one cipher at the right of the divisor, cut them off and as many places in the dividend.

5. Divide 1246 by 300.

3|00)12|46
——
4 Quotient, and 46 remainder.

6. Divide 84678 by 4400.

When the divisor consists of a composite number, divide first by one component part, and that quotient by the other.

7. Divide 844 by 24 (a composite number.)
Ans. 35 and 4 rem.

8. Divide 346 by 63 (a composite number.)
Ans. 5 and 31 rem.

Let the pupil understand why the remainders are 4 and 31 in the 7th and 8th examples.

9. Divide 459 by 35 (a composite number.)
Ans. 13 and 4 rem.

When the divisor is 1 with a cipher or ciphers, perform by cutting off as many places from the right of the dividend as there are ciphers in the divisor.

10. Divide 5678 by 100. Ans. 56 and 78 rem.
11. Divide 6485 by 1000. 12. Divide 1623 by 100.

Exercises on the preceding Rules.

1. A man had two farms, one contained 25 acres and the other 50. He bought another containing 135 acres. How many acres did he then have?

2. A man had a house, which cost $940, and a barn that cost $200. He built a shed that cost $155. What did these three buildings cost?

3. A man carried corn, rye and oats to market. He had 5 bushels of corn, which sold for 67 cents per bushel. He had 6 bushels of rye, which he sold for 80 cents per bushel. He had 10 bushels of oats, which he sold for 41 cents per bushel. What did he receive for all his grain?

4. A man died leaving an estate of $1000. He owed one man $12,50; another $12,50; and a third, $63. When his debts were paid, the remainder was divided among two sons and one daughter, giving each son twice as much as the daughter. What was the daughter's share, and each son's share?

Daughter's share $182,40.
Each son's share $364,80.

5. A man has one horse worth $85, and another $65. What is the difference between their values?

6. A man carried butter and cheese to market. He carried 40 pounds of butter, which he sold for 18 cents per pound. He carried 50 pounds of cheese, which he sold for 8 cents per pound. He bought 45 pounds of sugar at 9 cents per pound, and a barrel of flour for $5,75. How much was due him on settling?

Ans. $1,40.

7. A man hired two men and a boy to do a piece of work, and agreed to give them $2 for their work. The boy was to have fifty cents, and the remainder was to be divided in such a manner between the two men, as to give one man 15 cents more than the other. What was each man's share?

One man had $,825.
The other had $,675.

8. If 1 horse draw 1000 pounds to market, what will 6 horses draw, at the same rate?

9. If a man travel 37 miles in one day, how far, at that rate, will he travel in nine days?

10. If the earth move 68000 miles in one hour, how far will it move in a day? (24 hours.) How far in a year? (365.)

11. The thirteen United Colonies, which first adopted and issued the Declaration of Independence, entered into confederation in 1778, and formed the Constitution in 1780. Vermont was admitted into the Union in 1794. How long was the time between the confederation of the thirteen States and the admission of Vermont into the Union? How long, between the adoption of the constitution and the admission of Vermont into the Union?

Tennessee was admitted 1796. How long after the confederation? after the formation of the constitution? Kentucky was admitted 1798. How long after the confederation? formation of the Constitution? Ohio was admitteed in 1802. How long after the confederation and formation of the Constitution? Louisiana 1812. Indiana 1815. Mississippi 1817. Alabama 1818. Maine 1820. Missouri 1820. Arkansas 1836. Michigan 1837. How long from the confederaiion and for-

mation of the Constitution to the admission of each of these States?

§ 6. Parts of Units.

In the preceding sections, whole numbers have been considered. But it is often necessary to consider a value less than 1. Whenever a value is less than 1, it is a part of a unit, or 1. There are two methods of expressing the value of parts of units. The first is to write one figure above another, with a short line between them. From this form of expression, the second method is derived, by annexing ciphers to the upper figure, dividing by the lower and placing the quotient at the right of the point (,) as cents and mills are written. Thus,

First Method.		Second Method.
$\frac{1}{2}$	=	,5
$\frac{3}{4}$	=	,75
$\frac{3}{8}$	=	,375
$\frac{4}{5}$	=	,8

I. First Method.

The figure denoting the *number* of parts is written above the line, and is called the numerator. The figure denoting the *kind* of parts is written below the line, and is called denominator. The numerator answers to the dividend and the denominator, to the divisor. Each process of this method of parts may be analyzed, and the reason of each step readily discovered. To aid in the work, an arithmetical formula of analysis is given under each rule.

1. When the numerator and denominator are multiplied or divided by the same number, or numbers, the value of the expression is not altered. $\frac{4}{8}\div2=\frac{2}{4}\div2=\frac{1}{2}\times2=\frac{2}{4}\times2=\frac{4}{8}$.

2. By dividing the terms* of the parts, by any number, that will divide them both without a remainder,

---

* Terms of parts are numerator and denominator.

until no number greater than 1 will so divide them, the expression is changed into its lowest terms.

$\frac{16}{22} \div 2 = \frac{8}{16} \div 4 = \frac{2}{4} \div 2 = \frac{1}{4}$ Lowest terms.

Change into lowest terms $\frac{14}{112}$, $\frac{16}{144}$, $\frac{18}{721}$, $\frac{15}{105}$, $\frac{12}{120}$, $\frac{17}{102}$.

3. A whole number and parts ($4\frac{1}{2}$) is called a mixed number. A mixed number may be changed into the form of parts, by multiplying the whole number by the denominator, to that product adding the numerator, and writing the result over the denominator.

Thus, $4\frac{1}{2} = \frac{9}{2}$. $\frac{4}{1} \times 2 = \frac{8}{2} + \frac{1}{2} = \frac{9}{2}$ Ans.

Change into the form of parts, $5\frac{2}{3}$, $7\frac{4}{5}$, $8\frac{3}{7}$, $9\frac{7}{8}$, $8\frac{6}{11}$, $9\frac{1}{15}$.

4. When the numerator is equal to the denominator, or greater, the form is improper, and is to be changed into a proper one. This is done by dividing the numerator by the denominrtor. Thus, $\frac{9}{2} = 4\frac{1}{2}$.

Change to proper forms, $\frac{8}{4}$, $\frac{8}{5}$, $\frac{10}{3}$, $\frac{17}{5}$, $\frac{16}{3}$, $\frac{18}{4}$, $\frac{19}{6}$. As the lower number denotes the *kind* of parts, it shows how many of them make a unit, and by dividing the *number* of parts by that number which makes a unit, the quotient will be the number of units.

Change to proper forms $\frac{16}{2}$, $\frac{121}{11}$, $\frac{342}{121}$, $\frac{3687}{432}$.

5. To take a part of a whole number, multiply the whole number by the numerator, and divide the product by the denominator; or divide the whole number by the denominator and multiply the product by the numerator.

Take $\frac{3}{4}$ of 12. $12 \div 4 = 3$ (one fourth part of 12). $3 \times 3 = 9$ (three fourths of 12).

Take $\frac{6}{7}$ of 42. $\frac{8}{9}$ of 81. $\frac{6}{7}$ of 13. $\frac{8}{9}$ of 21.

6. To take a part of another part, multiply the numerators together for a new numerator and the denominators for a new denominator. Thus $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$. The value of an expression of parts is diminished, by multiplying the denominator. Hence to take $\frac{1}{2}$ of $\frac{1}{2}$, the denominator of $\frac{1}{2}$ is multiplied by 2. $\frac{1}{2} \times 2 = \frac{1}{4}$ (one half of $\frac{1}{2}$). And as only *one* half is taken, repeat $\frac{1}{4}$ *once*, or

multiply the numerators 1, 1, together, and we have the answer, $\frac{1}{4}$.

Take $\frac{5}{6}$ of $\frac{7}{8}$. $\frac{8}{9}$ of $\frac{10}{13}$. $\frac{6}{7}$ of $\frac{11}{14}$. $\frac{9}{10}$ of $\frac{10}{13}$. The same rule and method of analysis will apply to any number of expressions of parts.

Take $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{8}{9}$. $\frac{8}{9}\times_8=\frac{8}{72}$ (one eighth of $\frac{8}{9}$), $\frac{8}{72}\times^7=\frac{56}{72}$ (seven eighths of $\frac{8}{9}$), $\frac{56}{72}\times_4=\frac{56}{288}$ (one fourth of $\frac{7}{8}$ of $\frac{8}{9}$), $\frac{56}{288}\times^3=\frac{168}{288}$ (three fourts of $\frac{7}{8}$ of $\frac{8}{9}$), $\frac{168}{288}\div4=\frac{42}{72}\div6=\frac{7}{12}$ Ans. (in lowest terms.)

Take $\frac{6}{8}$ of $\frac{9}{11}$ of $\frac{4}{15}$. $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{16}{17}$ of $\frac{8}{9}$. $\frac{5}{8}$ of $\frac{6}{10}$ of $\frac{7}{10}$ of $\frac{11}{12}$. Take $\frac{3}{4}$ of $\frac{24}{5}$. $\frac{6}{7}$ of $8\frac{3}{7}$.

7. When a part is to be multiplied by a whole number, multiply the numerator by the whole number, and under the product write the denominator. (If the form be improper, change it. § 6. I. 4.)

Multiply $\frac{4}{9}$ by 2. $\frac{4}{9}\times^2=\frac{8}{9}$ Ans. Multiplying the *number* of parts, multiplies the value of the expression, if the *kind* of parts remain unchanged.

Multiply $\frac{5}{6}$ by 7. $\frac{7}{8}$ by 4. $\frac{8}{9}$ by 6. $\frac{9}{10}$ by 3. $\frac{10}{11}$ by $11\frac{3}{4}$. $\frac{8}{9}$ by $6\frac{3}{4}$.

8 When a part is to be divided by a whole number, divide the numerator or multiply the denominator, by the whole number. We have already said, that multiplying the denominator diminishes the value of an expression of parts. (§ 6. I. 6.) It is also evident, that, to diminish the *number* of parts (the *kind* remaining the same) diminishes the value of an expression of parts.

Divide $\frac{4}{9}$ by 4. $\frac{4}{9}\div^4=\frac{1}{9}$ Ans.
Divide $\frac{4}{5}$ by 3. $\frac{4}{5}\times_3=\frac{4}{15}$ Ans.
Divide $\frac{6}{7}$ by 8. $\frac{8}{9}$ by 10. $\frac{11}{12}$ by 9. $\frac{16}{24}$ by 11. $\frac{17}{25}$ by 6. $\frac{15}{18}$ by 13.

9. To find how many times a part is contained in a whole number, multiply the whole number by the denominator and divide the product by the numerator.

How many times are $\frac{3}{4}$ contained in 3? How many 4ths in 3? 3×4=12 (fourths). How many times are 3 fourths contained in 12 fourths? 12÷3=4 Ans.

How many times are $\frac{4}{5}$ contained in 4? $\frac{6}{7}$ in 3? $\frac{7}{8}$ in 10? $\frac{3}{8}$ 12?

10. To find how many times one part is contained in another part, invert the divisor (or part contained) and proceed as in § 6. I. 6.

How many times are $\frac{4}{5}$ contained in $\frac{6}{7}$?

How many times is 1 contained in $\frac{6}{7}$? $\frac{6}{7}$ (of 1) time.

How many times is $\frac{1}{5}$ contained in $\frac{6}{7}$? 5 times as many times as 1. (or $\frac{5}{5}$.) $\frac{6}{7}\times5=\frac{30}{7}$=the number of times that $\frac{1}{5}$ is contained in $\frac{6}{7}$. But as $\frac{4}{5}$ are 4 times as large a value as $\frac{1}{5}$, therefore, $\frac{4}{5}$ are contained in $\frac{6}{7}$, one fourth of the times that $\frac{1}{5}$ is found in $\frac{6}{7}$. Take $\frac{1}{4}$ of $\frac{30}{7}$ and we have the answer. $\frac{30}{7}\times_4=\frac{30}{28}=1\frac{1}{14}$.

How many times are $\frac{6}{8}$ contained in $\frac{8}{9}$? $\frac{7}{9}$ in $\frac{11}{12}$? $\frac{9}{13}$ in $\frac{17}{21}$? $\frac{18}{24}$ in $\frac{16}{17}$?

11. To unite parts, add the numerators, when the denominators are alike, and write the amount over the denominator.

Unite $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$. $2+3+4=9$. $\frac{9}{10}$ Ans.

Unite $\frac{2}{15}$, $\frac{3}{15}$, $\frac{4}{15}$, $\frac{5}{15}$. Unite $\frac{3}{21}$, $\frac{4}{21}$, $\frac{5}{21}$, $\frac{6}{21}$.

12. To unite parts when the denominators are unlike, change the expressions to those having like denominators, and then unite the numerators. Forms of parts may be changed to those having like denominators, by multiplying together all the denominators for a like denominator, and each numerator into all the denominators except its own for new numerators. The two sets of expressions are of the same value; this is seen from § 6. I. 1.

Unite $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{4}$ changed to expressions having like denominators and new numerators.. $\frac{40}{60}$ $\frac{48}{60}$ $\frac{45}{60}$. United $\frac{133}{60}=2\frac{13}{60}$ Ans.

Unite $\frac{3}{4}$, $\frac{6}{7}$, $\frac{8}{9}$. $\frac{6}{8}$, $\frac{9}{10}$, $\frac{11}{12}$. $\frac{6}{11}$, $\frac{6}{9}$, $\frac{8}{13}$. $\frac{10}{11}$, $\frac{9}{14}$, $\frac{5}{16}$, $\frac{6}{9}$.

13. To find the difference between parts, take the difference between the numerators, when the denominators are alike. If the denominators are unlike, change the expressions to those having like denominations, and then take the difference between the numerators.

Take $\frac{2}{3}$ from $\frac{5}{6}$. $\frac{15}{18}-\frac{12}{18}=\frac{3}{18}=\frac{1}{6}$ Ans.

Take $\frac{2}{3}$ from $\frac{3}{4}$. $\frac{3}{8}$ from $\frac{4}{5}$. $\frac{6}{11}$ from $\frac{8}{9}$. $\frac{5}{7}$ from $\frac{9}{10}$. $\frac{11}{130}$ from $16\frac{11}{23}$. $\frac{5}{75}$ from $3\frac{1}{16}$.

II. Second Method.

The relative value of parts of this method decreases from the left hand to the right, in a ratio of 10. This is seen in the following

SCALE.

Position.	,	Rate of decrease.
1st	×	Tenths
2d	×	Hundredths
3d	×	Thousandths
4th	×	Tenths of thousandths
5th	×	Hundredths of thousandths
6th	×	Millionths
7th	×	Tenths of millionths
8th	×	Hundredths of millionths
9th	×	Thousandths of millionths
10th	×	Tenths of thousandths of millionths
11th	×	Hundredths of thousandths of millionths
12th	×	Billionths
13th	×	Tenths of billionths
14th	×	Hundredths of billionths
15th	×	Thousandths of billionths
16th	×	Tenths of thousandths of billionths
17th	×	Hundredths of thousandths of billionths
81th	×	Trillionths

Examples for Reading.

,046
,606
,704678
,91216789
,000000000122
,000000001400144
,1216001610078109
,1678000000008881901
,2167801
,91234678916.

Examples for Writing.

Write in figures the following values.

One hundredth, Two tenths. Thirty three thousandths. One tenth of a thousandth. One hundredth of a billionth. Three hundredths of a millionth. Three hundredths of a thousandth of a billionth. Six trillionths.

1. To unite parts of this method, write them under each other according to their relative value, and proceed as in § 2.

Unite ,5 ,75 ,875.

,5
,75
,875

2,125 Amount.

Unite 1,25; 61,875; 125,03216; 234,01624.

Unite three hundredths and five thousandths of billionths.

Unite three tenths; One hundred, and three hundredths of billionths.

2. To find the difference between parts of this method, place the smaller value under the larger, according to their relative value, and proceed as in § 3.

Take ,46 from ,875.

,875
,46
,415 Difference.

Take ,375 from ,875. ,001 from 1. .005 from 5 ,505 from 5,05. ,6066 from 60,66. ,875 from 8,75.

3. To multiply parts of this method, proceed as in § 4,* and place the point where the relative value requires it. It is found, that there will always be as many places at the right of the point in the product, as there are at the right of the point in the multiplicand and multiplier. Thus, multiply ,5 by ,5. Doing this by the first method, we derive the following process.

$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=,25$. The same is obtained by calling ,5 a part of the first method, thus, $\frac{5}{10}\times\frac{5}{10}=\frac{25}{100}=\frac{1}{4}=,25$.

The same result is obtained, thus,

,5
,5
,25 Ans.

Multiply ,25 by ,5. ,875 by ,375. ,625 by ,75. ,22,375 by 10,625.

Multiply three thousandths by seven tenths of thousandths.

Multiply eight billionths by seven tenths of billionths.

4. To divide parts by parts, proceed as in § 5, and place the point in the quotient where the relative value requires it. Considering the dividend as a product, and the divisor as one factor and the quotient the other, it is plain, that there must be as many places at the right of the point in the divisor and quotient as there are in the dividend.

Divide 8,875 by 2.

2)8,8750
4,4825

---

* In this place, it is not necessary to write numbers under each other according to their relative value.

When a cipher is annexed to the dividend, it is counted as one place at the right of the point.

Divide 1,2 by 2. Ans. 6. Divide ,625 by 3.
Divide ,005 by ,05 Ans. ,1. Divide ,0099 by ,33.

When the dividend does not contain so many places at the right of the point, as there are in the divisor, make the number in the dividend equal to those in the divisor by annexing ciphers to the dividend.

Divide 99, by ,33.

```
,33(99,00(300 Ans.
    99
    ——
     00
```

Divide 90 by ,03. Divide 90,3 by ,003.
Divide 1 by ,1. Divide 10, by ,01.
Divide 05, by ,05. Divide 50 by ,50.

5. To change the form of a part in Second Method, to the form of a part in First Method. Write the denominator under the given part, which will always be 1 with as many ciphers at the right, as there are places at the right of the point in the given part.

Change ,5 to a part in First Method. Ans. $\frac{5}{10}$.
Change ,019 to First Method. Ans. $\frac{19}{1000}$.
Change ,25 to First Method. Ans. $\frac{25}{100}=\frac{1}{4}$ (lowest terms.)
Change ,75 to First Method. Ans. $\frac{3}{4}$.
Change ,875 to First Method. Ans. $\frac{7}{8}$.
Change ,625 to Frst Method.
Change ,125 ; ,15625 ; ,1875 ; ,28125 ; ,34375 ; ,40624 ; ,46875 . ,53125 to expressions of the First Method.

§ 7. Tables.

MONEY.

1 £ (pound) = 20*s.* (shillings.)
1*s.* = 12*d.* (pence.)
1*d.* = 4*qrs.* (farthings.)

$1 = 4*s* 6*d*. in England.
$1 = 5*s*. in Canada and Nova Scotia.
$1 = 6*s*. in New-England, Virginia, Kentucky and Tennessee.
$1 = 8*s*. in New-York, Ohio, and North Carolina.
$1 = 7*s*. 6*d*. in New-Jersey, Penn. Del. and Maryland.
$1 = 4*s*. 8*d*. in South Carolina and Georgia.

Troy Weight.

1lb. (pound) = 12oz. (ounces.)
1oz. = 20pwt. (penny weights.)
1pwt. = 24grs. (grains.)

Avoirdupois Weight.

1T. (ton.) = 20Cwt. (hundred weight.)
1cwt. = 4qrs. (quarters.)
1 qr, = 25lbs. (pounds.)
1lb. = 16oz. (ounces)
1oz. = 16drs. (drams)

Long Measure.

1 circle = 360° (degrees)
1° = 69½m. (miles)
1m. = 8 fur. (furlongs)
1fur. = 40rds. (rods)
1rd = 5½yds. (yards) or 16½ft. (feet)
3ft. = 1yd.
1ft. = 12in. (inches)
1in. = 3b. (barley corns)

Square Measure.

1m. (mile) = 640 a. (acres)
1a, = 4 roods,
1 rood = 40rds, (rods)
1rd. = 30¼yds, or 272¼ft.
9ft, = 1yd.
1ft = 144in.

CLOTH MEASURE.

1yd = 4qrs.
1qr. = 2e's (eighths)
1e. = 2n's. (nails)
1n. = 2¼in.

1 E. E. (Ell English) = 5 quarters.
1 E. F. (Ell Flemish) = 3 quarters.
1 E. Fr. (Ell French) = 6 quarters.
1 E. S. (Ell Scotch) = 37½ inches.

APOTHECARIES' WEIGHT.

1lb. (pound) = 12℥ (ounces.)
1℥ = 8ʒ (drams.)
1ʒ = 3℈ (scruples.)
1℈ = 20 grs. (grains.)

WINE MEASURE.

1 tun = 2 pipes.
1 pipe = 2hhds (hogsheads.)
1hhd. = 63galls. (gallons.)
42galls. = 1 tierce.
1gal. = 4qts. (quarts)
1qt. = 2pts. (pints.)
1pt. = 4 gills.

DRY MEASURE.

1bu. (bushel) = 4pks. (pecks,)
1pk. = 8qts. (quarts.)
1qt. = 2 pints.

CIRCULAR MEASURE.

1 circle = 12s. (signs.)
1s. = 30° (degrees.)
1° = 60′ (minutes.)
1′ = 60″ (seconds)

MILK AND BEER MEASURE.

1 tun = 2 butts.
1 butt = 2hhds.
1hhd. = 54gals.
36gals. = 1 barrel.
1gal. = 4qts.
1qt. = 2 pints.

TIME.

1y. (year) = 12c.m. (calendar months) or one Julian year = 13m. 1d. 6 hours, or 365d. 6h.
1c.m. = 4w. (weeks.)
1w. = 7d. (days.)
1d. = 24h. (hours.)
1h. = 60m. (minutes.)
1m. = 60s.(sec'ds.)

Calendar Months in their order, with the number of days in each.

1.	January	has	31	days.
2.	February	"	28	" (and in leap year 29.)
3.	March	"	31	"
4.	April	"	30	"
5.	May	"	31	"
6.	June	"	30	"
7.	July	"	31	"
8.	August	"	31	"
9.	September	"	30	"
10.	October	"	31	"
11,	November	"	30	"
12.	December	"	31	"

§ 8. To change compound values to simple ones, and simple, to compound.

I. To change compound values to simple ones of *the smallest kind*, multiply the largest kind by as many units of the next smaller kind, as make 1 unit of the largest kind, and to that product add the value of the next smaller kind, (if their be any value of that kind expressed.) Proceed in this manner, until all the values be changed into the kind required.

1. Change £4 10s. 6d. 2qrs. into farthings.

```
   4 10 6 2
  20
  --
  90
  12
----
1086
   4
----
4346 Ans.
```

2. Change £3 8s. 6½d. into farthings. Ans. 3290.
3. Change 3lbs. 8oz. 15pwts. 18grs. into grains.
4. Change 7 T. 12cwt. 3qrs. 18lbs. 12oz. 11drs. into drams.
5. Change 3lbs. 8℥, 5ʒ. 2℈. 10grs. into grains.
6. Change 2m. 3fur. 20 rods, 10 feet, 8in. 2b. into barley corns.
8. Change 1y. 8c.m. 2w. 3d. 10h. 40m. 30s. into seconds.
8. Change 4bu. 3pks. 5qts. 1pt. into pints.
9. Change 5yds. 1e. into nails.
10. Change $4^\circ\ 5'\ 6''$ into seconds.
11. Change 1a. 2 roods, 5yds. 5ft. 108in. into inches.
12. Change 1 C. 1yd. 1ft. into inches.
13. Change one year into seconds.
14. Tell how many months you have lived.
15. Tell how many minutes you have lived.
16. Tell how many seconds you have lived.

II. To change compound values into simple ones of *the largest kind*, annex ciphers to the lowest kind and divide it by as many units of that kind, as make 1 unit of the next higher kind, and to the higher kind annex the quotient. Proceed with the next higher kind in the same manner, and so with each kind, until all the values be changed into the kind required.

1. Change £4 8s. 6d. 3qrs. into the simple value of pounds.

4)3,00
12)6,75
2,0)8,5625
Ans. £4,428125

2. Change £3 6s. 4d. 2qrs. into pounds.
3. Change 3lbs. 8oz. 15pwts. 18grs. into pounds.
4. Change 7 T. 12cwt. 3qrs. 18lbs. 12oz. 11grs. into tons
5. Change 1y. 8c.m. 2w. 3d. 10h. 40m. 30s. into years.
6. Change 5yds. 2qrs. 2 nails, into yards.
7. Change 4° 5′ 6″ into degrees.

III. To change compound values into any required simple value. If there be larger kinds, than the required simple, change them into the required simple, as in § 8. I. If there be smaller kinds, than the required simple, change them into the required simple, as in § 8. II.

1. Change £1 17s. 11d. 1qr. into the value of pence.

$1\times20=20+17=37\times12=444+11=455,25d.$ Ans.

2. Change £2 8s. 6d. 3qrs. into the value of shillings.
3. Change 7lbs. 10oz. 14pwt. 20grs. into the value of ounces.
4. Change 4bu. 3pks. 4qts. 1 pint into the value of pecks.

5. Change 4lbs. 6℥. 4ʒ. 1℈. 8grs. into the value of ounces.

6. Change 1 C. 8yds. 16ft. into the value of solid yards.

7. Change 4yds. 2qrs. 1e. 1na. into the value of eighths.

IV. To change a simple value of the *largest kind*, expressed *as a part* in Second Method to compound values of lower kinds, or units of a simple value of a lower kind. Multiply the value (expressed in parts, or at the right of the point,) by as many units of the next lower kind, as make 1 unit of the given value, and point off the product as the value requires. (§ 6. II. 3.)

1. Change £4,6875 to compound values of lower kinds.

$$\begin{array}{rl} ,6875 & \text{(figures at the right of the point.)} \\ 20 & \\ \hline 13,7500 & \\ 12 & \\ \hline 9,0000 & \end{array}$$

13s. 9d. required values.

Ans. £4 13s. 9d. = £4,6875.

2. Change £5,6625 to compound values of lower kinds.

3. Change £ ,8 to units of a lower kind.

4. Change £4,9875 to compound values of lower kinds.

5. Change ,975 of a tun of wine to compound values of lower kinds.

6. Change 3,615 cwt. to compound values of lower kinds.

V. To change a simple value expressed *as a part* in First Method to compound values of lower kinds, or to units of a simple value of a lower kind. Multiply the part given by as many units of the next lower kind, as make 1 unit of the value given. (§ 6. I. 7.) If the product contain units and a part, or a part only, multiply that part by as many units of the next lower kind, as make 1 unit of the kind of the part. So continue with each successive part, until the value be changed into the required values.

1. Change £$\frac{5}{6}$, into compound values of lower kinds.

$\frac{5}{6}\times{}^{20}=\frac{100}{6}=16\frac{4}{6}$. $\frac{4}{6}\times{}^{12}=\frac{48}{6}=8$. Ans. 16s. 8d.

2. Change £$\frac{4}{7}$ to compound values of lower kinds.

3. Change $\frac{9}{16}$ of an acre to compound values of lower kinds.

4. Change $\frac{5}{13}$ of a ton to compound values of lower kinds.

5. Change $\frac{7}{8}$ of a pound Avoirdupois to compound values of lower kinds.

6. Change $\frac{8}{9}$ of a yard to compound values of lower kinds.

VI. To change a simple value to compound values of larger kinds. Divide the given value by as many units of the same kind as make 1 unit of the next larger value. Divide the quotient by as many units of the same kind as make 1 unit of the next larger kind. Proceed in this manner until the required values be obtained.

1. Change 24648 farthings into the compound values of pounds, shillings and pence.

4)24648

12)6162 6

20)513 13

25 Ans. £25 13s. 6d.

2. Change 84648 farthings into pounds, shillings and pence.

3. Change 11520 grs. (Troy Weight) into compound values of larger kinds.

4. Change 1024000 drs. to higher kinds.
5. Change 144in. into higher kinds in Cloth Measure.
6. Change 63115200 seconds into higher kinds in Time.

§ 9. To find how many times one value, simple, or compound, is contained in another value, simple, or compound.

Change all the compound values to the same simple value; (§ 8. III.) and proceed as in § 5.

1. How many times are 6s. contained in £1 10s. 6d.? Change the compound values into the simple value of shillings and divide by 6s.

£1 10s. 6d. = 30, 5s. ÷ 6 = 5,08½ Ans.

2. In $25, how many pounds Canada money? That is, how many times is one pound contained in $25?

Change £1 and $25 to shillings, Canada money, and divide by the value of £1.

$25 × 5s. (see Table) = 125 ÷ 20 = £6 5s.

3. How many times are 8s. contained in £4 10s. 8d. 3qrs.?
4. How many times are 7s. 6d. contained in £2 8s. 6d.?
5. How many times are 4s. 6d. contained in £1 10s. 6d.?
6. How many times are 4s. 8d. contained in £3 8s. 4d.?
7. How many times are 4s. 6d. Georgia currency, contained in $500?
8. How many times are 4s. 6d. English money, contained in $450?
9. How many times are 7s. 6d. Penn. currency, contained in $325?
10. How many times are 2 feet 8 inches contained in 1 mile? in 4 furlongs? in 8 rods?
11. How many times are 16½ feet contained in 44 feet? in 4 rods and 11 feet? three furlongs, 20 rods, and 10 feet?

12. How many times are 8 hours and 20 minutes contained in 1 year and 4c.m.?

13. How many times are 3pks. and 2qts. contained in 4bu. 3pks. and 1 pint?

14. How many times is 1 penny, English money, contained in $3?

§ 10. To change improper forms of compound values to proper ones.

An improper form of a compound value consists in having as many units in one value, as make 1 unit of the next larger value, or more units than make 1 unit of a larger value. Thus, £4 20s. 14d. 4qrs. is an improper form of expressing the value contained in those kinds.

To change an improper form of this kind, divide the smallest kind by as many units of that kind, as make 1 unit of the next higher kind, and unite the quotient with the next higher kind. Proceed in the same manner with each kind, until the expression be changed to a proper form.

1. Change £4 20s. 14d. 4qrs. to a proper form.

```
  4)4—0 rem.
    —
    1
   14
 12)15—3 rem.
    —
    1
   20
    —
 20)21—1 rem.
    —
    1        Ans. £1 1s. 3d. 0qrs.
```

2. Change £5 24s. 16d. 8qrs. to a proper form.

3. Change £6 26s. 18d. 9qrs. to a proper form.

4. Change 1y. 14c.m. 6w. 14d. 60h. 20m. 1400s. to a proper form.

5. Change 4 miles, 10fur. 80 rods, 33ft. 24in. to a proper form.

§ 11. Uniting Compound Values.

To unite compound values, unite the value of each kind, (§ 2.) and then change the form (if it be improper) to a proper one.

1. Unite £4 15s. 10d. 3qrs.; £5 16s. 8d. 2qrs. £6 18s. 11d. 3qrs.

£4 15s. 10d. 3qrs.
£5 16. 8d. 2qrs.
£6 18s. 11d. 3qrs.

£15 49s. 29d. 8qrs.

Changed to a proper form. Ans. £17 11s. 7d. 0qrs.

2. Unite £20 16s. 11d. 3qrs.; £16 16s. 8d. 2qrs.; £13 13s. 10d. 3qrs.

3. Unite 10 miles, 7fur. 11ft. 10in.; 8m. 6fur. 39 rods, 11ft. 11in.

4. Unite 10y. 8c.m. 3w. 6d.; 6y. 9c.m. 2w. 5d.

5. Unite 4 T. 3cwt. 2qrs. 14lbs. 10oz. 11drs.; 3 T. 4cwt. 3qrs. 16lb. 11oz. 12drs.

6. Unite £ $\frac{1}{16}$, $\frac{4}{11}$s. and $\frac{4}{5}$d.

7. Unite £2 $\frac{6}{11}$s. 8d. and 1qr.

§ 12. Finding the difference between Compound Values.

To find the difference between compound values, place the less value under the greater, and take the lower from the upper. If the value in any kind in the lower number be greater than the one above it, add as many units to the upper value as make 1 unit of the next larger kind, and from that amount take the lower value. In this case, add 1 to the next higher value in the lower number, before taking it from the value above it.

1. What is the difference between £4 8s. 6d. 2qrs. and £3 6s. 9d. 1qr.

4 8 6 2
3 6 9 1

Aans. £1 1s. 9d. 1qr.

2. What is the difference between 7lbs. 4oz. 14pwt. 20grs., and 3lbs. 8oz. 10pwt. 18grs. ?
3. What is the difference between 60y. 9c.m. 2w. 1d. 13h. 25m. 22s., and 25y. 8c.m. 3w. 5d. 15h. 23s. ?
4. Find the difference between £$\frac{2}{6}$ and $\frac{2}{8}$s.
5. Find the difference between £6 $\frac{8}{11}$.

To find the difference between dates, reckon 30 days in a month and 12 months in a year.

6. Find the difference between Oct 10th 1824, and Oct. 6th 1834.

1834 9 6
1824 9 10

Ans. 9 11 26

Reckon *to* the month, in getting the number of months to be set down.

7. Find the difference between Sept. 11, 1624, and June 8, 1715.
8. Find the difference between 1844 April 23, and 1845 June 16.
9. Find the difference between 16° 8′ 22″ East Longitude, and 24° 4′ 14″ E. L.
10. Find the difference between 14° 6′ 15″ E. L. and 14° 8″ 14″ West Longitude.

§ 13. Repeating Compound Values.

To repeat compound values, proceed as in § 4, and change the form of the result (if it be improper) to a proper one. (§ 10.)

1. Multiply £8 8s. 10d. 3qrs. by 4.

$$\begin{array}{cccc} 8 & 8 & 10 & 3 \\ & & & 4 \\ \hline \end{array}$$

£32 32s. 40d. 12qrs. = £33 15s. 7d. 0qr. Ans.

2. Multiply £2 10s. $11\frac{6}{2}$d. by 12.
3. Multiply £3 8s. 6d. 3qrs. by 9.
4. Multiply 2m. 7fur. 33 rods, 11ft. 3in. 2b. by 8.
5. Multiply £$\frac{5}{7}$, $\frac{5}{6}$s. $\frac{2}{11}$d. by 4.
6. Multiply 5yds. 3qrs. 1e. by 6.

§ 14. Dividing Compound Values.

To divide compound values, divide the largest kind; and the remainder (if there be any) change to, and unite it with, the next lower value, (§ 8. I.) and divide that result. Proceed in this manner, until each kind be divided. If the divisor be not contained in any given kind, change that kind to the next lower, and so on, until the divisor be contained in the given value.

1. Divide £3 10s. 6d. 2qrs. by 2.

$$\begin{array}{r|cccc} 2) & 3 & 10 & 6 & 2 \\ \hline & £1 & 15s. & 3d. & 1qr. \end{array}$$ Quotient.

2. Divide 6lbs. 10oz. 14pwt. 16grs. by 8.
3. Divide 14 T. 15cwt. 3qrs. 14lbs. 12oz. 11drs. by 20.
4. Divide 16yrs. 8c.m. 3w. 5d. 8h. 20m. 30s. by 11.
5. Divide 11s. 24° 15′ 16″ by 18.

EXERCISES.

1. In 16248 in. how many furlongs, rods, &c?

2. In 5323500 drams (Avoirdupois Weight) how many tons, cwt. &c.?

3. In 51672 pwt. how many pounds, &c.?

4. Change 54862 grains to compound values of higher kinds.

5 In 8lbs. 7℥. 2ʒ. 1℈. 18grs. how many grains?

6. In £42 15s. 8d. how many farthings,

7. In 13 tons, 14cwt. 1qr. 18lbs. 7oz. 11dr. how many drams?

8. What will 6cwt. 2qrs. 16lbs. of sugar cost, at 14 cents per pound?

9. In 678yds. 1qr. 1e. 1na. how many nails?

10. In 221 ells Eng. how many yards?

11. How many times are 16 feet 6in. contained in 40 miles?

12. In 15y. 18d. 12h. 36m. 46s. how many seconds?

13. In £45 4s. English money, how many dollars?

14. In £6 11s. 10d. 2qrs. Canada money, how many dollars?

15. In £3 12s. 8d. 3qrs. New-York money, how many dollars?

16. In £15 8s. 6d. Penn. money, how many dollars?

17. In 1 penny Eng. money, what part of a dollar?

18. In 1 farthing Eng. money, what part of a dollar?

19. In 1 farthing New-England money, what part of a dollar?

20. What will 55 boxes of sugar cost, worth £4 11s. 6d. 3qrs. per box?

21. What will 35 yards of cloth cost, at 5s. 6½d. per yard?

22. What will 24 barrels of flour cost at £1 6s. Eng, money per barrel?

23. How many cwt. in 234 barrels of flour, each barrel weighing 1cwt. 3qrs. 9lbs.?

24. If 426 yards of cloth cost £94 10s. 4½d., what is the price of one yard?

25. If 426bu. 3pks. 4qts. 1pt. be divided equally among 145 persons, how much will each one receive?

§ 15. Use of Money.

Law fixes the highest price to be paid for the use of money. In New-England this price is $6 for the use of $100 for one year. The number of dollars paid for the use of a hundred is called *rate per cent.*, or *rate by the hundred.* The money paid for the use of any sum hired is called Interest. The sum hired is called Principal. The Principal and Interest added make the Amount. At six per cent. the interest on any sum for one year is found, by taking $\frac{6}{100}$ of the Principal.

1. What is the interest of $156 for one year at 6 per cent.?

$$156\times6=936\div100=\$9{,}36 \text{ Ans.}$$

As a general rule, to find the interest on any sum of money for one year at 6 per cent. take

$$\frac{6}{100}=\frac{1}{16\frac{2}{3}}=,06 \text{ of the Principal.}$$

2. What is the interest of $200 for one year?

$$200\times,06=\$12{,}00 \text{ Ans.}$$
$$200\div16\tfrac{2}{3}=\$12{,}00 \text{ Ans.}$$
$$200\times6\div100=\$12{,}00 \text{ Ans.}$$

As $\frac{6}{100}$ represents the part of the Principal to be taken as interest, for one *year*, the part to be taken as interest for one month will be $\frac{1}{12}$ of that part.

Thus, $\frac{1}{12}$ of $\frac{6}{100}=\frac{6}{1200}=\frac{1}{200}$ the part of the Principal to be taken as interest for one month.

3. What is the interest of $200 for one month?

$$\$200\div200=\$1, \text{ Ans.}$$

As a general rule, to find the interest on any sum for one month, take $\frac{1}{200}=,005$ of the Principal.

4. What is the interest of $600 for one month?

$$600 \div 200 = \$3,\ \text{Ans.}$$
$$600 \times ,005 = \$3,000\ \text{Ans.}$$

To find the interest on any sum of money for any number of months. First find the interest for one month, and multiply that interest by the number of months.

5. What is the interest of $480 for 7 months?

$$480 \div 200 = \$2,40 \times 7 = \$16,80.$$

To find the interest for days, call them so many 30ths of a month.

6. What is the interest of $480 for 7 months and 7 days?

$$480 \div 200 = \$2,40 \times 7\tfrac{7}{30} = \$17,36.$$

7. What is the interest of $675 for 9 months and 9 days?

As $\frac{1}{200}$ denotes the part of the Principal to be taken as interest for one *month*, $\frac{1}{30}$ of this part will show what part of the Principal is to be taken as interest for one day. Thus, $\frac{1}{30}$ of $\frac{1}{200} = \frac{1}{6000}$ the part of the Principal to be taken as interest for one day.

To find the interest on any sum at 6 per cent. for one day, take $\frac{1}{6000} = ,00016\frac{2}{3}$ of the Principal.

8. What is the interest of $480 for one day at 6 per cent.?

$$\$480 \div 6000 = \$,08.$$

9. What is the interest of $640 for one day at 6 per cent.?

To find the interest on any sum for any number of days at 6 per cent., first find the interest for one day, and multiply that interest by the number of days.

10. What is the interest of $425,625 for 1 year and 8 months at 6 per cent.?

11. What is the interest of $376,26 for 2 years, 6 months and 9 days at 6 per cent.?

When the rate is any other than 6 per cent., the figure showing how many dollars are paid for the use of 100 for one year, will be the numerator and 100 the denominator, expressing the part of the Principal to be taken as interest at that rate. Thus, what part of the Principal is to be taken as interest for one year at 1 per cent.? Ans. $\frac{1}{100}$. at 2 per cent.? Ans. $\frac{2}{100}$. at 3 per cent.? Ans. $\frac{3}{100}$. at 4 per cent.? Ans. $\frac{4}{100}$. at 5 per cent.? Ans. $\frac{5}{100}$, and so on, for any required rate.

Hence, as a general rule, to find the interest on any sum at any rate per cent. Take that part of the Principal, indicated by the rate per cent.

12. What is the interest of $200 for one year at 1 per cent.?

$$200\times1=200\div100=\$2.$$

13. What is the interest of $600 for one year at $6\frac{1}{2}$ per cent.?

$\frac{6\frac{1}{2}}{100}=\frac{13}{200}$ the part of the Principal to be taken as interest.

$$600\times13=7800\div200=\$39.$$

14. What is the interest of $650 for one year at 5 per cent.? at $4\frac{1}{4}$ per cent.? at $7\frac{1}{2}$ per cent.? at $9\frac{3}{4}$ per cent.? at $8\frac{3}{4}$ per cent.?

When any rate per cent. other than six per cent., is required for any other time, than one year, first find the interest for the given time at 6 per cent., and add to that interest, or take from it, so much as the interest sought may require.

15. What is the interest of $200 for 4 months at 2 per cent.?

$$200\div200=1\times4=\$4 \text{ (interest at 6 per cent.)}$$

To get 2 per cent. from 6 per cent.—take out 4 per cent. from 6 per cent. Or, take $\frac{1}{3}$ of 6 per cent.

$$4\div3=\$1,33\tfrac{1}{3} \text{ Ans.}$$

16. What is the interest of $150 for one year and six months at $4\frac{1}{2}$ per cent.? at $5\frac{1}{2}$ per cent.? at $7\frac{3}{4}$ per cent.?

17. What is the interest of \$480 for 1y. 6m. 13d. at 7 per cent.? at 8 per cent.? at $9\frac{3}{4}$ per cent.?

18. What is the interest of \$500 for 2y. 5m. 19d. at 6 per cent.?

§ 16. Proportional Values.

I. The quotient arising from dividing one value by another is called the ratio between the divisor and dividend. Hence, the ratio between any two values may be found, by dividing one by the other. Thus, the ratio between 4 and 8 is 2.

$8\div 4=2$ ratio. These terms 4 and 8 are expressed, thus, 4:8 when their relation to each other is intended. The first term 4 is called antecedent, and the second, 8 is called consequent. Two terms are necessary to give rise to a ratio. Four terms give rise to proportion. Thus, 4:8::6:12, read thus, 4 is to 8, as 6 to 12. These four terms contain two ratios. There is a ratio between 4 and 8, and also one between 6 and 12. And in all proportional values, these two ratios are the same. Hence, proportion is an equality of two or more ratios. Knowing the ratio of two numbers, in a proportion, and having one term necessary for a second ratio, we can easily find the fourth term in proportion.

Thus, 4:8::6: read thus, 4 is to 8 as 6 to a required number. The ratio between 4 & 8 is 2. $8\div 4=2$. Consequently the fourth term is found by multiplying 6 by 2. $6\times 2=12$ the required term. The proportion it stated thus,

$$4 : 8 :: 6 : 12.$$

The same result may be obtained by another course of reasoning. In proportional values, the product of the first and last terms is equal to that of the second and third. Hence, dividing the product of the second and third te[illegible] by the first, the quotient will be the fourth term.

4 : 8 : : 6

8× [illegible] 48÷[illegible] =12 fourth term.

In performing problems in proportional values, either of the above methods may be adopted, for the reasons there given.

1. If 60 acres of land cost $2520, what will 100 acres cost?

60 : 100 : : 2520

$2520\times100=252000\div60=$4200 Ans.

It [illegible] be remembered, that, in order to obtain the ratio between any two values, those values must be of the *same kind.* And whenever two values, that give rise to a ratio, are given in different kinds, they must first be changed to values of the same kind, and then their ratio taken.

2. If 25 acres of land cost $250, what will 12 acres and 80 rods of land cost?

40)80

4)2 roods

,5 a.

25 : 12,5 : : 250
12,5

1250
500
250

25)3125,0(125.
25

62
50

125
125

Ans. $125.

3. If 12 yards of cloth cost $50, what will 5ds. 2qrs. 1e. and 1n. cost?

4. If 128 feet of wood, or one cord, cost $5, what will 75 feet and 87 inches cost?

5. If a man pay $6,25 for a horse and wagon to go 10 miles, how much, must he pay, at the same rate, to go 7 miles, 14 rods, 15 feet, and 11¼ inches?

II. To find the Present worth of any given sum, for any specified time. Present worth is a sum, which, if put at interest, will in a given time, amount to a certain sum.

As the amount of one dollar for the given time and rate is to one dollar, so is the given sum to the present worth.

1. What is the present worth of $127,20 due one year hence, discounting at the rate of 6 per cent.?

1,06 : 1,00 : : 127,20 : $120 Ans.

2. What is the present worth of $426,47, due 9 months hence, discounting at the rate of 4 per cent.? at 5 per cent.? at 6½ per cent.?

3. What is the present worth of $560, due 6 months hence, discounting at the rate of 6 per cent.?

The discount is found by subtracting the present worth from the given sum.

4. What is the discount on 480 dollars for 1y. 6m. at 5 per cent.? at 7 per cent.? at 8 per cent.?

5. What is the discount on $365 for 3y. 8m. at 6 per cent.? at 8 per cent.?

III. LOSS AND GAIN.

1. To find what is gained or lost per cent. Find the gain or loss, then as the cost is to the gain or loss, so is 100 to the gain or loss per cent

1.) If broadcloth cost $4 per yard, and is sold for $4,50 per yard, what is the gain per cent.?

4,50
4,00
,50 gain per yard.
4 : ,50 : : 100
,50
4)50,00
12,50

Ans. $12\frac{1}{2}$ per cent.

2.) If butter is bought for $12\frac{1}{2}$ cents per pound and sold for 15, how much is gained per cent.?

,15
,125
,025 gain per pound,
,$12\frac{1}{2}$: ,$02\frac{1}{2}$: : 100 : 20

Ans. 20 per cent.

3.) If cheese is bought for $6\frac{1}{4}$ cents per pound and sold for $8\frac{1}{3}$, what is gained per cent.?

4.) If beef is bought for 5 cents per pound and sold for $4\frac{3}{4}$, what is lost per cent.?

5.) If sugar is bought at $6\frac{1}{2}$d. per lb. and is sold at £2 4s. 6d. per cwt. what is lost per cent.?

2. When the price, at which an article is sold, and the gain or loss per cent. are known, to find the cost at which the article was bought.

To find the cost, when there is a gain.

As 100 with per cent. added, is to 100, so is the price to the first cost.

1.) If broadcloth is sold for $4,50 per yard, and there is gained $12\frac{1}{2}$ per cent., what did the cloth cost per yard?

$112\frac{1}{2}$: 100 : : $4,50 : $4 first cost.

2.) If butter is sold for 15 cents per pound, and there is gained 20 per cent., what was the cost per pound?

3.) If cheese is sold for 9 cents per pound, and there is gained 10 per cent., what was the cost of one pound?

To find the cost, when there is a loss.

As 100 with the per cent subtracted, is to 100, so is the price to the first cost.

4.) If cotton cloth is sold for 15 cents per yard, and there is lost 7½ per cent., what was the first cost?

$$100 - 7\tfrac{1}{2} = 92\tfrac{1}{2}$$

$$92\tfrac{1}{2} : 100 :: ,15 : ,162\tfrac{6}{37}.$$

Ans. $,162\frac{6}{37}$.

5.) If butter is sold for 12½ cents per pound, & there is lost 6¼ per cent., what was the first cost?

6.) If broadcloth is sold for $5,00 per yard, and there is lost 10 per cent., what was the first cost?

7.) If a man sell a horse for $75, and lose 50 per cent., what was the cost of the horse?

3. When the cost of an article is known, to find at what price it must be sold to gain or lose a certain per cent.

To find the price, when there is to be a certain given per cent.

As 100 is to 100 with the per cent. added, so is the cost of the article to the required price.

1.) If cloth is bought at $3,00 per yard, how must it be sold, to gain 25 per cent.?

$$100 : 125 :: \$3 : \$3,75$$

2.) If bread is bought for 3 cents per pound, how must it be sold to gain 37½ per cent.? Ans. $,04⅛.

3.) If butter is bought for 10 cents per pound, how must it be sold to gain 50 per cent.?

To find the price, when there is to be a certain loss per cent.

As 100 is to 100 with the loss subtracted, so is the cost of the article to the required price.

4.) If cheese is bought for 6¼ cents per pound, how must it be sold to lose 3 per cent.?

100 : 97 : : ,0625 : ,060625.

Ans. 6 cents and ⅝ of a mill.

5.) If lard is bought for 10 cents per pound, how must it be sold to lose 25 per cent.? 12½ per cent.? 18¾ per cent,? 33⅓ per cent.?

IV. Principal, interest, and time given to find the rate per cent.

As the interest at one per cent. on the given principal, is to the given interest, so is 100 to the rate per cent.

1.) Principal $120, interest, $13,20, and time 16 months given to find the rate per cent.

$1,60 : $13,20 : : 100 : 6¼. Ans. 8¼ per cent.

2.) Principal $126, interest $17,01, time 18 months given to find the rate per cent. Ans. 9 per cent.

3.) Principal $130, interest $12,1875, time 15 months given to find the rate per cent. Ans. 7½.

V. Principal, interest, and rate per cent. given, to find the time.

As the interest, at the given rate per cent. for one year on the given principal, is to the given interest, so is one year to the required time.

1.) Principal $150, interest $4,50, rate per cent. 6, given to find the time.

$9 : $4,50 : : 12 : 6 Ans. 6 months.

2.) Principal $450, interest $27, rate per cent. 9, given to find the time. Ans. 8 months.

3.) Principal $200, interest 8, rate per cent 6, given to find the time. Ans. 8 months.

VI. Interest, time, and rate per cent. given to find the principal.

As the interest, at the given rate per cent. for the given time, on one dollar, is to the given interest, so is one dollar to the principal required.

1.) Interest $31,50, time 14 months, rate per cent. 6, given to find the principal.

,07 : 31,50 : : 1 : 450 Ans. $450.

2.) Interest $27,00, time 1y. 6c.m., rate per cent. 3, given to find the principal. Ans. $600.

3.) Interest $175, time 1 year, rate per cent. $\frac{1}{2}$, given to find the principal. Ans. $350.

4.) Interest $,12$\frac{1}{2}$, time 4 months, rate per cent. $\frac{1}{4}$, gvien to find the principal. Ans. $150.

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